Application Note 1009

Understanding Measurements using an Oscilloscope versus a Lock In Amplifier

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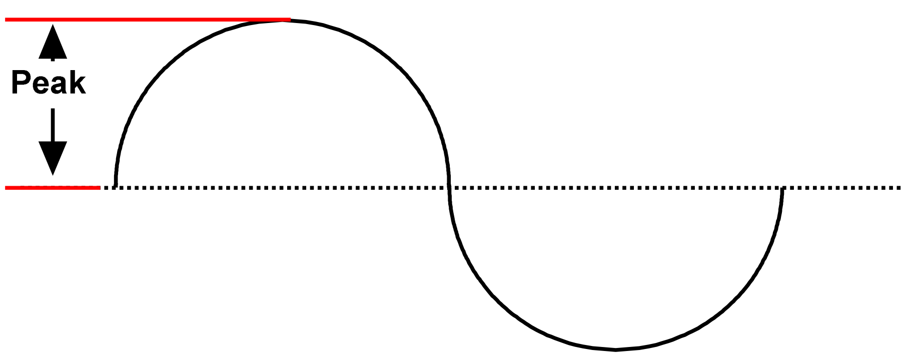
Introduction

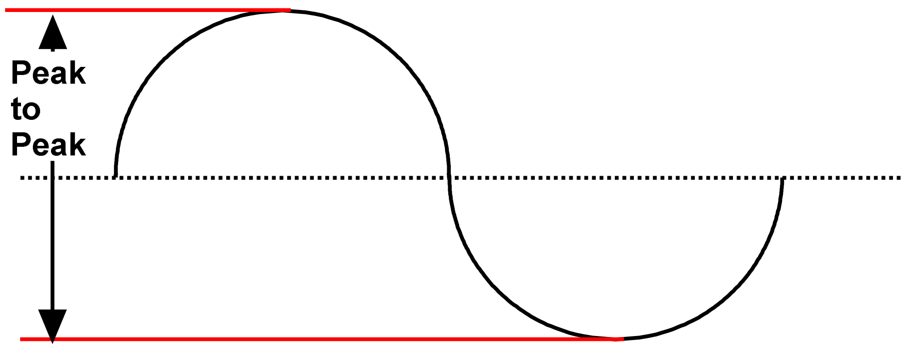
At Spectrum Detector, we calibrate our sensors for use with an oscilloscope. All our sensors have responsivity in Volts per Watt. When using a sensor with an oscilloscope, the power can be obtained by dividing the peak to peak voltage by the responsivity. When measuring very small light levels, an oscilloscope may not be able to provide a good measurement. The instrument of choice in these conditions is a Lock In Amplifier (LIA). When using an LIA, the readings it gives will not match those given by an oscilloscope. The reason for this is that the LIA measures the RMS value of the waveform, while the oscilloscope measures the peak to peak value. To make measurements with the LIA, a conversion factor must be found. Note that some digital scopes have an RMS measurement function, but the accuracy suffers along with the peak to peak measurement as the signal level drops. As will be shown, the RMS reading from a scope will not always match that measured by an LIA.

AC Waveforms

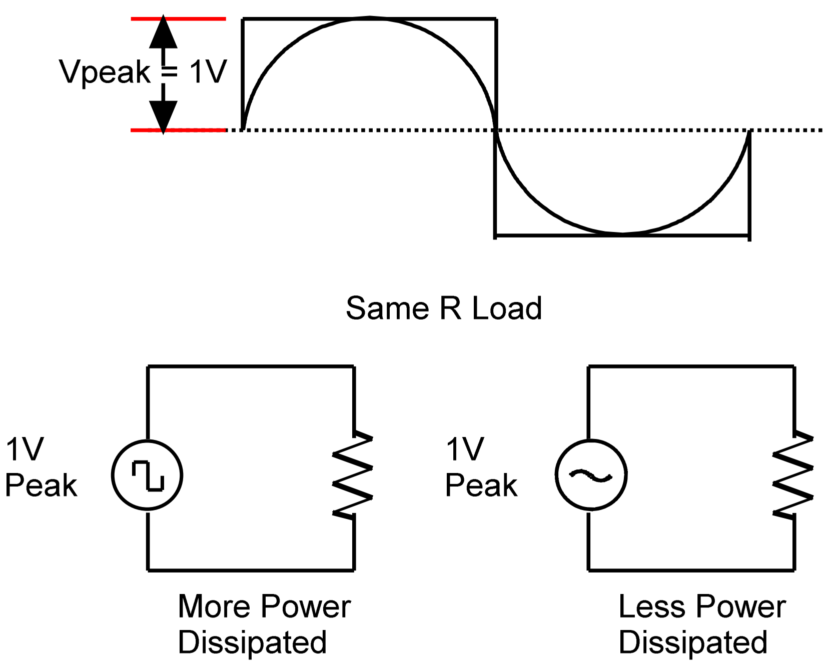
AC voltage alternates in polarity and AC current alternates in direction. We have a problem when we try to define the amplitude of an AC signal. With DC, measuring how much voltage or current we have is easy as the values are stable. Since AC signals are constantly changing, what do we measure? One way to express the amplitude of an AC quantity is to measure its maximum positive voltage on an oscilloscope. This is known as the peak value of an AC waveform.



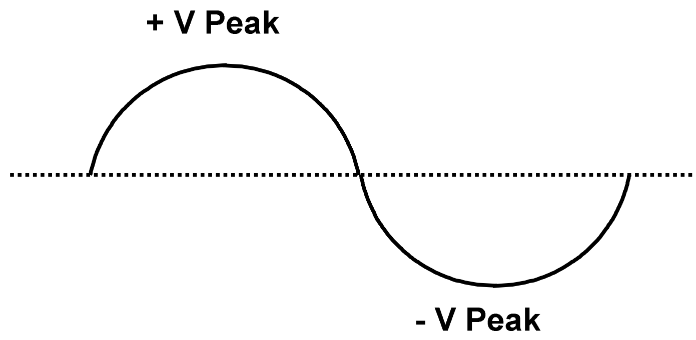
Another way is to measure the total voltage between opposite peaks. This is known as the peak-to-peak (P-P) value of an AC waveform.



These values for waveform amplitude can be confusing when comparing different types of waves. A square wave peaking at 1 volt is a greater amount of voltage for a greater amount of time than a sine wave peaking at 1 volt. The effects of these two AC voltages powering a load would be quite different.

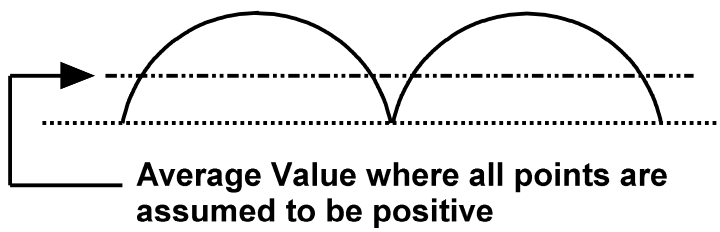


One way of expressing the amplitude of different waveforms is to average the values of all the points on a waveform to a single number. This amplitude measure is the average value of the waveform. The average value for ground centered symmetric waveforms is zero, because all the positive points cancel out all the negative points over a full cycle.

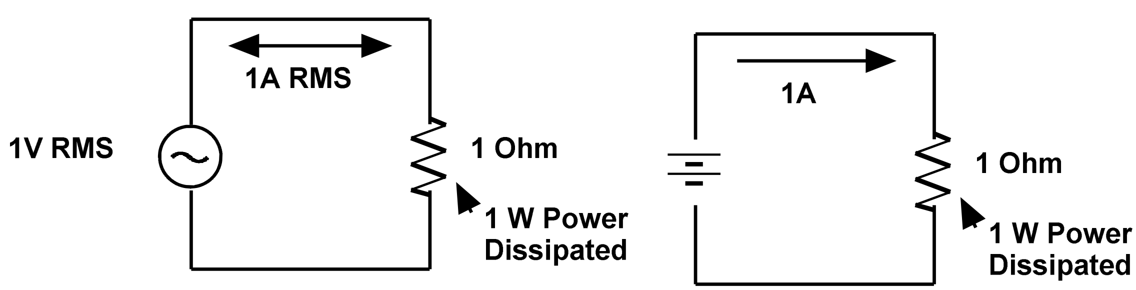


Average Value of all points is zero

A better measure of a waveform's average value is defined as the average of all the absolute values over a cycle. In other words, we calculate the average value of the waveform by taking the absolute value of all points, as if the waveform looked like this.



The best method of deriving a value for AC waveform amplitude is based on the waveform's ability to do work when driving a load resistance. An AC measurement based on work performed by a waveform is not the same as that waveform's average value. The power dissipated by a given load is not proportional to the voltage or current. Power is proportional to the square of the voltage or current. The idea is to find a DC equivalent to any AC voltage or current. We want to find the DC voltage or current that would produce the same amount of power dissipated through an equal resistance.



Equal Power Dissipated by equal loads

An RMS voltage produces the same heating effect as the same DC voltage. In the two circuits above, we have the same amount of load resistance (1 Ω) dissipating the same amount of power in the form of heat (1Watt). Because the AC voltage source is equivalent in terms of power delivered to a load to a 1 volt DC battery, we say its voltage value is 1 volt RMS. RMS stands for Root Mean Square. Peak and peak-to-peak measurements are best performed with an oscilloscope, which can capture the peaks of the waveform with a high degree of accuracy. For low amplitude signals in the presence of noise and interference, the LIA is the best instrument to use. The LIA isolates the fundamental sine wave component of the signal and measures its RMS value. The fundamental sine wave component is simply a sine wave at the same frequency of the measured waveform. It is this fundamental sine wave isolation the enables the LIA to measure signals in the presence of noise and interference that would swamp an oscilloscope. It essentially homes in on the fundamental and ignores everything else. A calibration factor then converts the measurement to the RMS value.

Converting From Peak to RMS

For simple waveforms, conversion factors can be calculated for equating Peak, Peak-to-Peak, Average, and RMS measurements to one another. The equation for the RMS value of a waveform is:

$$V_{RMS} = \left[\frac{1}{T} \cdot \int_0^T v(t)^2 \, dt \right]^{\frac{1}{2}}$$

Table 1 shows the relationships derived from this equation between peak, peak to peak, average, and rms values.

Waveform	V peak	V peak to peak	V average	V rms
Sine	1	2	0.637	0.707
Square (50% Duty Cycle)	1	2	1	1
Triangle	1	2	0.5	0.577

Table 1

The peak to peak, peak, and RMS values of these waveforms were measured with a Tektronix TDS460 oscilloscope and a Stanford Research 510 LIA. Table 2 shows the results.

Waveform	TDS Vpp	TDS V peak	TDS RMS	Stanford Research RMS
Sine	200mV	100mV	70.6mV	70.5mV
Square	200mV	100mV	99.7mV	89.5mV
Triangle	200mV	100mV	57.8mV	57.1mV

Table 2

Note that the measured RMS values for the sine and triangle wave are in good agreement with table 1, but the square wave is not. This is due to the fact that the LIA isolates the fundamental frequency component of the waveform. For a sine wave, the fundamental and the waveform are identical. The frequency content of a square wave contains many more frequency components that were filtered out by the LIA.

Why is the triangle waveform error smaller? The Fourier expansion for a 50% duty cycle square wave of 1V peak repeating at 100Hz is:

$$V\ Square = \frac{4}{\pi}\cdot 1\ V_{peak}\ \sin\ at\ 100Hz + \frac{4}{\pi}\cdot\frac{1}{3}V_{peak}\ \sin\ at\ 300Hz + \frac{4}{\pi}\cdot\frac{1}{5}\ V_{peak}\ \sin\ at\ 500Hz$$

$$+\frac{4}{\pi}\cdot\frac{1}{7}\ V_{peak}\ \sin\ at\ 700Hz + \frac{4}{\pi}\cdot\frac{1}{9}V_{peak}\ \sin\ at\ 900Hz + ...\quad goes\ on\ forever$$

So a square wave of 100mV peak at 100Hz will have a sine wave component at 100Hz with amplitude of 100mV times 4 divided by pi. A pure sine wave of 100mV peak at 100Hz will have a component with amplitude of 100mV at 100Hz. The LIA is calibrated to read this sine wave as 70.7mV, that is it is calibrated to read out in RMS. Therefore the square wave fundamental will be read out as:

$$V\ Square = 100mV\cdot\frac{4}{\pi}\cdot 0.707 = 90mV_{RMS}$$

The Fourier expansion for a triangle wave of 1V peak repeating at 100Hz is:

$$V\ Triangle = \frac{8}{\pi^2}\cdot 1V_{peak}\ \sin\ at\ 100Hz - \frac{8}{\pi^2}\cdot\frac{1}{9}V_{peak}\ \sin\ at\ 300Hz + \frac{8}{\pi^2}\cdot\frac{1}{25}V_{peak}\ \sin\ at\ 500Hz$$

$$-\frac{8}{\pi^2}\cdot\frac{1}{49}V_{peak}\ \sin\ at\ 700Hz + \frac{8}{\pi^2}\cdot\frac{1}{81}V_{peak}\ \sin\ at\ 900Hz + ...\quad goes\ on\ forever$$

The triangle’s harmonics are divided by the square of n, whereas the square wave’s harmonics are divided by n. The harmonics of the triangle are not as crucial as they are much lower in amplitude. The amplitude of the fundamental will be read out as:

$$V\ Triangle = 100mV\cdot\frac{8}{\pi^2}\cdot 0.707 = 57.3mV_{RMS}$$

These values agree well with the measured ones.

Example

A signal is measured with an LIA using an SPH-42 detector. with an Rv of 20,000 V/W. The detector is being used at 100Hz, so it is out of the flat band response and its output voltage is a triangle waveform. The Rv of the detector at 100Hz is measured and found to be 2100V/W for a peak to peak waveform. The measured RMS value from the LIA is 23.6mV. We must convert the peak to peak Rv to a peak Rv, (divide by 2) then to an RMS Rv (multiply by 0.573). The measured power will then be:

$$Power = \frac{23.6mV}{\frac{0.573}{2} \cdot 2{,}100\frac{V}{W}} = 39.2\mu W$$

Conclusion

When using an oscilloscope to measure the peak to peak voltage of Spectrum Detector’s sensors, the corresponding power can be easily found by dividing the peak to peak reading from the oscilloscope by the responsivity of the sensor. When using a Lock In Amplifier, the responsivity must be scaled to represent the RMS value of the fundamental component of the waveform. For the waveforms commonly encountered when using Spectrum Detector’s sensors, table 3 lists the scale factors.

Waveform	Scope	LIA
Square (50% Duty Cycle)	Use Standard Rv	Divide Rv by 2 And Multiply by 0.9
Triangle	Use Standard Rv	Divide Rv by 2 And Multiply by 0.573

Table 3